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الثالث -A-

"Numerical Analysis"

1- why use Numerical methods??

1. To solve mathemtical problems that cannot be solved exacttly
2. To solve engineering problems by digital computers.
3. For "Real_Time" engineering Computing.

2- steps in solving an engineering problem by numerical methods.

1. problem definition
2. Mathematical Model
3. choice of numerical method
4. programming and operation
5. interpretation of results

3-

* Accuracy: How close is a computed or measured value to the true value.
* precision: How close a computed or measured value to previously computed or measured values.
* Inaccuracy: A systematic deviation from the actual value
* Imprecision: Magnitude of scatter.

4- Sources of Error

① Round off error
② Truncation error
③ Experimental
④ Programming

* Round off error (له علاقة بالجزء الذي تقبله الحاسبة)

① caused by representing a number approximately by significant number of digits

② computation always done with fixed significant digits.

* There are Two ~~type~~ Methods

chopping ... and rounding

التقطيع (قطع الكسر بغض النظر عن قيمة الجزء المقطوع)

$\pi = 3.14159265$

Chopping $= 3.141592$

عند التقريب يجب مراعاة الجزء المراد قطعه

$\pi = 3.14159265$

Rounding $= 3.141593$

* Truncation error تعريف

Error caused by truncating or approximating a mathematical procedure.

تعتمد على العمليات الحسابية التي تستخدمها الحاسبة

Why measure error?

1. To determine the accuracy of numerical results.
2. To develop stopping criteria for iterative algorithm

* الخطأ الحقيقي True Error: the difference between the true value in acalculation and the approximate value found using anumerical method.

True Error = True Value − Approximate Value

* Relative True Error الخطأ الحقيقي النسبي

the ratio between the true error and the true value.

* Relative Approximate Error

the ratio between the approximate error and the present approximation

5- What is Root?

In engineering it is frequently have to find Solution of equations in the form

$$f(x) = 0$$

ex

$$x^2 - 3x + 2 = 0 \Rightarrow \#$$

* There are many Methods to find Roots of equation:

① Netwon's Raphsan Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

x_i = (تعطى) القيمة الاصلية

x_{i+1} = تمثل القيمة الجديدة

② SeCand Method

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

to find

③ Bisection Method

* في هذه الطريقة نعطى قيمتان لـ x

x_{lower} and x_{upper}

* نستخرج x_m الجديدة من القانون

$$x_m = \frac{x_L + x_u}{2}$$

لتكملة الحل هناك ثلاث طرق فإذا كانت :

① $f(x_L)\,f(x_m) < 0$ يعني قيمة سالبة

$x_u = x_m \quad , \quad x_L = x_L$

② $f(x_L)\,f(x_m) > 0$ يعني قيمة موجبة

$x_L = x_m \quad , \quad x_u = x_u$

③ $f(x_L)\,f(x_m) = 0$ $x_m = root$

وهو الجذر المطلوب

Taylor Series

* $f(x_{i+1}) = f(x_i) + f'(x_i)\,h + \frac{f''(x_i)\,h^2}{2} + \frac{f'''(x_i)\,h^3}{3!} + \text{—}$ ①

* $f(x_{i-1}) = f(x_i) - f'(x_i)\,h + \frac{f''(x_i)\,h^2}{2!} - \frac{f'''(x_i)\,h^3}{3!} + ----$

* $f(x_{i+1}) = f(x_{i-1}) + 2f'(x_i)\,h + 2\frac{f'''(x_i)\,h^3}{3!} + ----$

* $f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$ forward

* $f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$ backward

* $f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$ Centered

* $\epsilon = \frac{\text{exact} - \text{error}}{\text{exact}}$ *

Example :- for Bisection Method

$$f(x) = x^3 - 5x + 1$$

$x_l = 0$
$x_u = 1$

	x_l	x_u	x_m	$f(x_l)$	$f(x_u)$	$f(x$
1	0	1	0.5	1	-3	-1..
2	0	0.5	0.25	1	-1.375	-0.2
3	0	0.25	0.125	1	-0.2343	0.37
4	0.125	0.25	0.1875	0.3769	-0.2343	0.06
5	0.1875	0.25	0.21875	0.0691	-0.2343	0.07

① $x_m = \dfrac{x_l + x_u}{2} = \dfrac{0+1}{2} = 0.5$

$f(x_l) = 0 - 0 + 1 = 1$

$f(x_u) = 1 - 5 + 1 = -3$

$f(x_m) = (0.5)^3 - 5(0.5) + 1 = -1.375$

- $f(x_l)\ f(x_m) < 0$

$1 * -1.375 = -1.375 < 0$

$\therefore x_l = x_l, \quad x_u = x_m$

② $x_m = \dfrac{0.5+0}{2} = 0.25$

$f(x_l) = 1$

$f(x_m) = (0.25)^3 - 5(0.25) + 1$

$= -0.2343$

$f(x_l)\ f(x_m) = -0.2343 < 0$

$\therefore x_u = x_m \ \& \ x_l = x_l$

$x_u = 0.25, \quad x_l = 0$

③ $x_m = \dfrac{0.25+0}{2} = 0.125$

$f(x_m) = (0.125)^3 - 5(0.125) + 1$

$= 0.3769$

④ $f(x_l)\ f(x_m) = 0.3769 > 0$

$\therefore x_l = x_m = 0.125$

$x_u = 0.25$

$f(x_m) = (0.1875)^3 - 5(0.1875) + 1$

$= 0.0691$

$f(x_l) = (0.125)^3 - 5(0.125) + 1$

$= 0.3769$

$f(x_l)\ f(x_m) = 0.0260 > 0$

$\therefore x_u = x_u, \quad x_l = x_m$

⑤ $x_m = \dfrac{0.1875 + 0.25}{2}$

Secant Method

$f(x) = \cos x = x$ $\qquad x_{i-1} = 0.5$

$f(x) = x - \cos x$ $\qquad x_i = 1$

$f(x) = x_i - \cos x \Rightarrow f(x_i) = 1 - \cos 1 = 0.45969$

$f(x_{i-1}) = -\cos 0.5 + 0.5 = -0.37758$

$$x_{i+1} = x_i - \frac{f(x_i) * (x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

x_{i-1} —— x_i —— x_{i+1}

$$= 1 - \frac{0.45969 * (1 - 0.5)}{0.45969 + 0.37758} = 0.72548$$

$$x_{i+1} = 0.72548 - \frac{f(x_i) * (x_i - x_{i-1})}{f x_i - f(x_{i-1})}$$

$$x_{i+1} = 0.72548 - \frac{[-0.02270 * -0.27452]}{[-0.02270 - 0.45969]}$$

x_i	x_{i-1}	x_{i+1}	$f(x_i)$	$f(x_{i-1})$	ϵ
1	0.5	0.72548	0.45969	-0.37758	
0.72548	1	0.73839	-0.02270	0.45969	0..
0.73839	0.72548	0.73778	-0.00116	0.02270	
0.73776	0.73839	0.738173	-0.00291	0.00116	

م في حالة Sin & Cos تحول الحاسبة الى "RAD"

$5f(x) = \cos x,$ $\quad$ 3-1 $\quad$ $f(x_i) = \cos \frac{\pi}{4}$ $\quad$ $f(x_{i+1}) = \cos \pi$

4.2

5
3-1

$f(x) = \cos x, \quad x_i = \frac{\pi}{4}, \; x_{i+1} = \frac{\pi}{3}, \; h = \frac{\pi}{12}$

$F(x_i) = \cos\frac{\pi}{4} = 0.707106781$

$F(x_{i+1}) = \cos\frac{\pi}{3} = 0.5$

$\epsilon = \frac{0.5 - 0.707106781}{0.5} \times 100 = -41.4$

n	$f^{n}(x)$	$F(\frac{\pi}{3})$	ϵ
0	$\cos x$	0.707106781	-41.4
1	$-\sin x$	0.521986658	-4.4
2	$-\cos x$	0.497754491	
3	$\sin x$	0.499869147	
4	$\cos x$	0.500007551	
5	$-\sin x$	0.500000304	
6	$-\cos x$	0.499999987	

① $F(x_{i+1}) = F(\frac{\pi}{3}) = \cos\frac{\pi}{4} + f'(x_i)h$

$= \cos\frac{\pi}{4} - \sin(\frac{\pi}{4})(\frac{\pi}{12}) = 0.521986658$

$\epsilon = \frac{\text{exact} - \text{error}}{\text{exact}} = \frac{0.5 - 0.521986658}{0.5} = -4.4$

② $F(2) = \cos\frac{\pi}{4} - \sin(\frac{\pi}{4})(\frac{\pi}{12}) - \cos\frac{\frac{\pi}{4}(\frac{\pi}{12})^2}{2} = 0.497754491$

$\epsilon = \frac{0.5 - 0.49775449}{0.5} = 0.00449$

③ $F(3) = \cos\frac{\pi}{4} - \sin\frac{\pi}{4}(\frac{\pi}{12}) - \frac{\cos(\frac{\pi}{4})(\frac{\pi}{12})^2}{2} + \frac{\sin(\frac{\pi}{4})(\frac{\pi}{12})^3}{6} = 0.4998691$

$\epsilon = \frac{0.5 - 0.499869147}{0.5} = 0.00026$

④ $f(4) = \cos(\frac{\pi}{4}) - \sin(\frac{\pi}{4})(\frac{\pi}{12}) - \frac{\cos(\frac{\pi}{4})(\frac{\pi}{12})^2}{2} + \frac{\sin(\frac{\pi}{4})(\frac{\pi}{12})^3}{6} + \frac{\cos(\frac{\pi}{4})(\frac{\pi}{12})^4}{24} = 0.500007551$

$\epsilon = \frac{0.5 - 0.500007551}{0.5} = -0.0000151$

⑤ $F(5) = \cos\frac{\pi}{4} - \sin\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right) - \frac{\cos\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^2}{2} + \frac{\sin\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^3}{6}$

$+ \frac{\cos\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^4}{24} - \frac{\sin\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^5}{120}$

$= 0.500000304$

$\epsilon = \frac{0.5 - 0.500000304}{0.5} = -0.00000608$

⑥ $f(6) = \cos\frac{\pi}{4} - \sin\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right) - \frac{\cos\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^2}{2} + \frac{\sin\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^3}{6}$

$+ \frac{\cos\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^4}{24} - \frac{\sin\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^5}{120} - \frac{\cos\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^6}{720}$

$= 0.499999987$

$\epsilon = \frac{0.5 - 0.499999987}{0.5}$

$= 0.000000026$